FIFTH SHORT COURSE

SOIL DYNAMICS IN ENGINEERING PRACTICE

WHEELING,IL APRIL 29-30, 2013

SOIL DYNAMICS AND MODELING INCLUDING WAVE PROPGATION AND DAMAGE TO STRUCTURES.

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(updated March 2013)

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SOIL DYNAMICS AND MODELING

1. SOURCE OF DYNAMIC LOADING
2. WAVE PROPAGATION
3. DAMAGE DURING EARTHQUAKE
4. IDEALIZATION OF SOILS AND STRUCTURES FOR ANALYSIS
5. VIBRATION ANALYSIS

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1. SOURCE OF DYNAMIC LOADING

- EARTHQUAKES
  Seismology, Epicenter-distance, Focal depth, Magnitude
  Modified Mercalli (MM) Scale
- PEAK GROUND MOTION
  H- and V- Ground motion
- FREQUENCY CONTENT
- RESPONSE OF HUMANS AND STRUCTURES TO VIBRATIONS
- DESIGN SEISMIC COEFFICIENTS

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SOURCES OF CONSTRUCTION VIBRATIONS

(ORGANIZED BY TYPE)

- TYPES OF VIBRATIONS
  - Transient or Impact
    Blasting, Impact Pile Driving, Demolition.
  - Steady State (Continuous)
    Vibratory Pile Driver, Large Pumps, Compressors.
  - Pseudo Steady State
    Jack Hammers, Pavement Breakers, Trucks, etc.

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- **TYPICAL WAVE PROPOGATION CURVES**

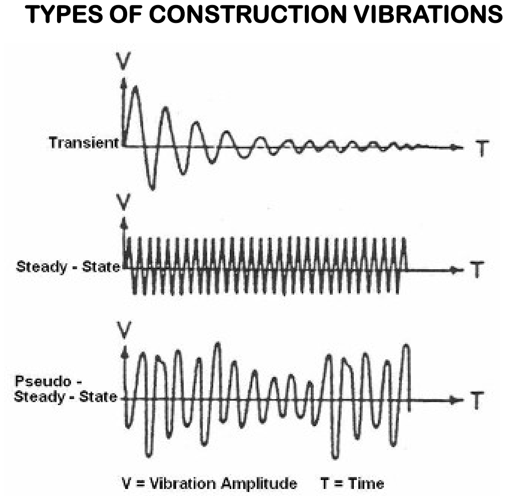

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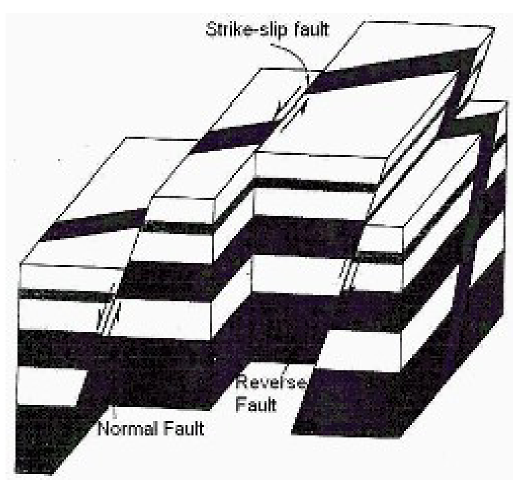
DIAGRAM SHOWING THE THREE MAIN TYPES OF FAULT MOTION



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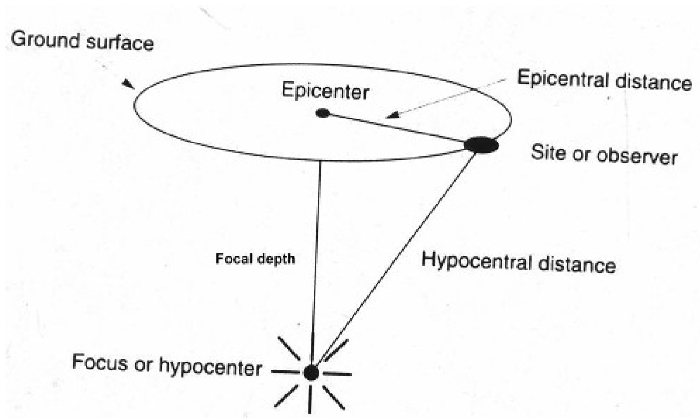
NOTATION FOR DESCRIPTION OF EARTHQUAKE LOCATION



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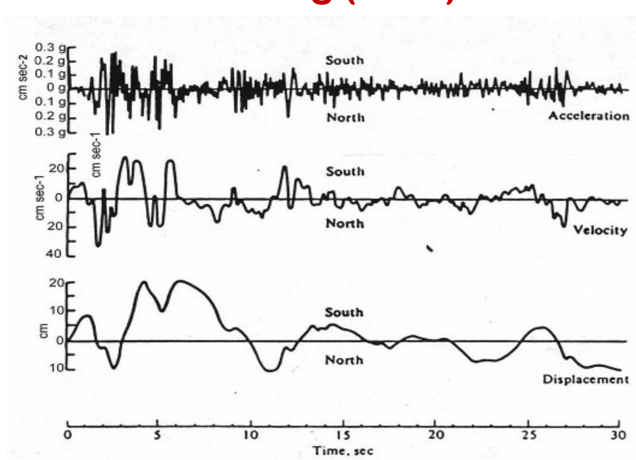
El Centro, Calif., earthquake of May 18, 1940, N-S component.

Source: J.A. Blume, N.M. Newmark, and L.H. Corning (1961).



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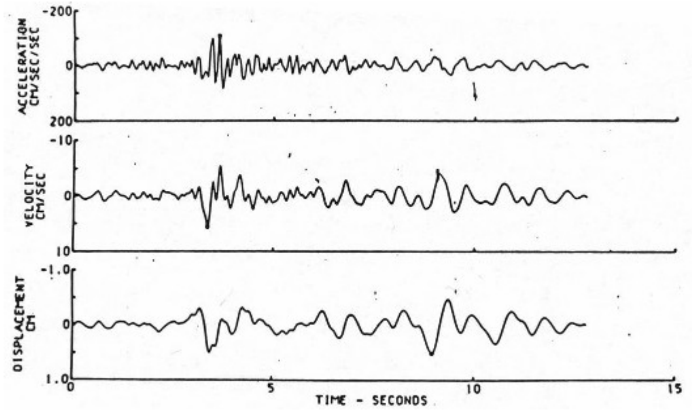
CORRECTED ACCELERATION, VELOCITY, DISPLACEMENT (M8.3) GUATEMALA EARTHQUAKES BETWEEN FEBRUARY 21 AND MAY 26, 1976 – SHOCK 1 CHICHICASTENANGO. SOUTH COMP PEAK VALUES ACCEL = -110.1 CM/SEC/SEC., VELOCITY = 5.635 CM/SEC., DISPL = 0.516 CM



International Symposium on the February 4th , 1976, Guatemalan earthquake and the reconstruction process. "Guatemalan strong-motion earthquake records" C.F. Knudson, V. Perez

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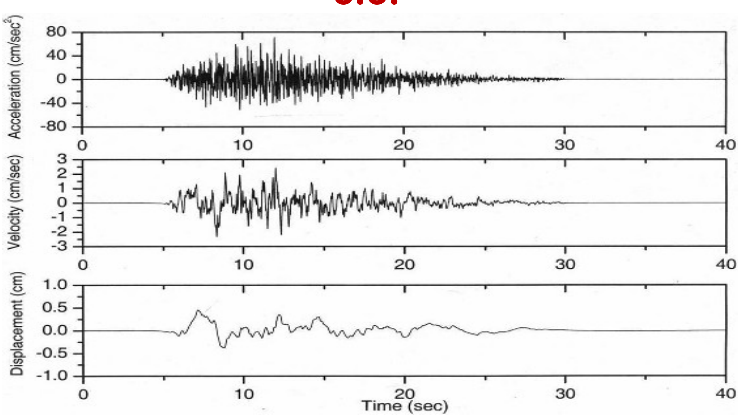
Memphis *Synthetic* Ground Motion, Magnitude 6.3.



Source. Wen, Y.K., Wu, C.L., 2000, "Generation of Ground Motions for Mid-America Cities.

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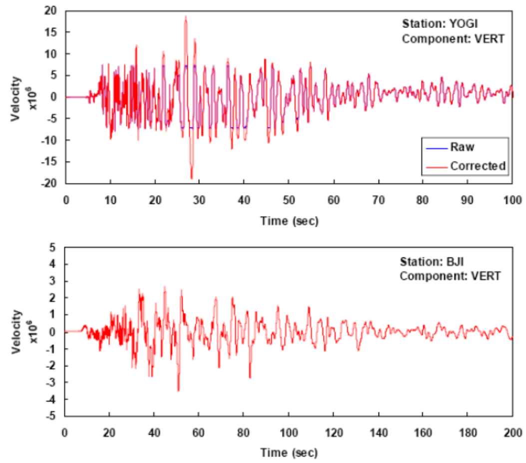
Vertical velocity data at station YOGI and BJI.



(Elnashai et al. 2006) (Indonesia, May 27, 2006)

Richter Local Magnitude

In 1935, Charles Richter used a Wood – Anderson seismometer to define a magnitude scale for shallow, local (epicentral distances less than about 600 km (375 miles)) earthquakes in southern California (Richter, 1935).

Richter defined what is now known as the *local magnitude* as the logarithm (base 10) of the maximum trace amplitude (in micrometers) recorded on a Wood – Anderson seismometer located 100 km (62 miles) from the epicenter of the earthquake.

The Richter local magnitude (ML) is the best known magnitude scale, but it is not always the most appropriate scale for description of earthquake magnitude.

Surface Wave Magnitude

The Richter local magnitude does not distinguish between different types of waves. Other magnitudes scales that base the magnitude on the amplitude of a particular wave have been developed. At large epicentral distances, body waves have usually been attenuated and scattered sufficiently that the resulting motion is dominated by surface waves. The *surface wave magnitude* (Gutenberg and Richter, 1936) is a worldwide magnitude scale based on the amplitude of *Rayleigh waves* with a period of about 20 sec. The surface wave magnitude is obtained from:

$$Ms = \log A + 1.66 \log\Delta + 2.0$$

Where,

A = maximum ground displacement in micrometers;

Δ **= epicentral distance of the seismometer measured in degrees. 360 degrees corresponding to the circumference of the earth.**

Note that the surface wave magnitude is based on the maximum ground displacement amplitude (rather than the maximum trace amplitude of a particular seismograph); therefore, it can be determined from any type of seismograph. The surface wave magnitude is most commonly used to describe the size of shallow (less than about 70 km (44 miles) focal depth), distant (farther than about 1000 km (622 miles)) moderate to large earthquakes.

(Geotechnical Earthquake Engineering 1996, Steven L. Kramer)

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Body Wave Magnitude

For deep-focus earthquakes, surface waves are often too small to permit reliable evaluation of the surface wave magnitude. The *body wave magnitude* (Gutenberg, 1945) is a worldwide magnitude scale based on the amplitude of the first few cycles of *p-waves* which are not strongly influenced by the focal depth (Bolt, 1989). The body wave magnitude can be expressed as

$$Mb = \log A - \log T + 0.01\Delta + 5.9$$

Where,

A = p-wave amplitude in micrometers;

Δ **= epicentral distance of the seismometer measured in degrees. 360 degrees corresponding to the circumference of the earth;**

T = period of the p-wave (usually about one sec). Body wave magnitude can also be estimated from the amplitude of one-second-period, higher-mode Rayleigh waves (Nuttli, 1973); the resulting magnitude, MbLg, is commonly used to describe intraplate earthquakes.

(Geotechnical Earthquake Engineering 1996, Steven L. Kramer)

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MODIFIED MERCALLI INTENSITY SCALE OF 1931

I	Not felt except by a very few under especially favorable circumstances
II	Felt by only a few persons at rest, especially on upper floors of buildings; delicately suspended objects may swing
III	Felt quite noticeably indoors, especially on upper floors of buildings, but many people do not recognize it as an earthquake; standing motor cars may rock slightly; vibration like passing of truck; duration estimated
IV	During the day felt indoors by many, outdoors by few; at night some awakened; dishes, windows, doors disturbed; walls make cracking sound; sensation like heavy truck striking building; standing motor cars rocked noticeably
V	Felt by nearly everyone, many awakened; some dishes, windows, etc., broken; a few instances of cracked plaster; unstable objects overturned; disturbances of trees, piles, and other tall objects sometimes noticed; pendulum clocks may stop
VI	Felt by all, many frightened and run outdoors; some heavy furniture moved; a few instances of fallen plaster or damaged chimneys; damage slight

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Cont., MODIFIED MERCALLI INTENSITY SCALE OF 1931

VII	Everybody runs outdoors; damage negligible in buildings of good design and construction, slight to moderate in well-built ordinary structures, considerable in poorly built or badly designed structures; some chimneys broken; noticed by persons driving motor cars
VIII	Damage slight in specially designed structures, considerable in ordinary substantial buildings, with partial collapse, great in poorly built structures; panel walls thrown out of frame structures; fall of chimneys, factory stacks, columns, monuments, walls; heavy furniture overturned; sand and mud ejected in small amounts; changes in well water; persons driving motor cars disturbed
IX	Damage considerable in specially designed structures; well-designed frame structures thrown out of plumb; great in substantial buildings, with partial collapse; buildings shifted off foundations; ground cracked conspicuously; underground pipes broken
X	Some well-built wooden structures destroyed; most masonry and frame structures destroyed with foundations; ground badly cracked; rails bent; landslides considerable from river banks and steep slopes; shifted sand and mud; water splashed over banks
XI	Few, if any (masonry) structures remain standing; bridges destroyed; broad fissures in ground; underground pipelines completely out of service; earth slumps and land slips in soft ground; rails bent greatly
XII	Damage total; practically all works of contruction are damaged greatly or destroyed; waves seen on ground surface; lines of sight and level are distorted; objects thrown into the air

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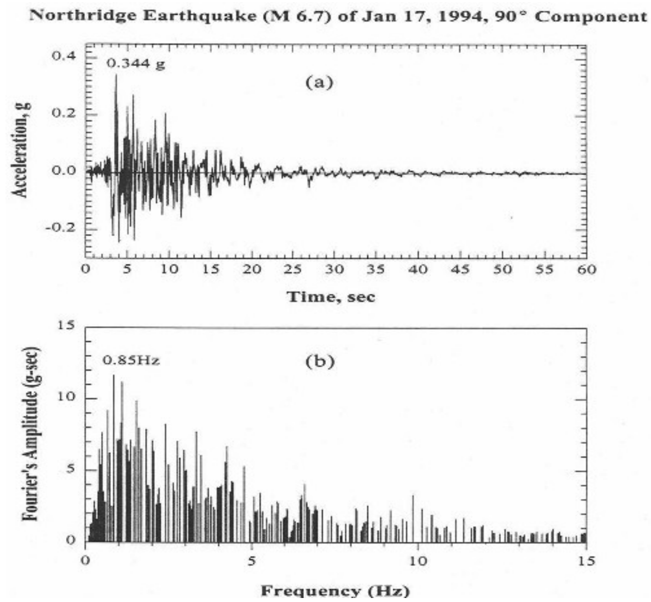
Northridge earthquake of Jan 17, 1994, 90 degree component (M6.7): a) accelerogram, b) Fourier's spectrum with predominant frequency.



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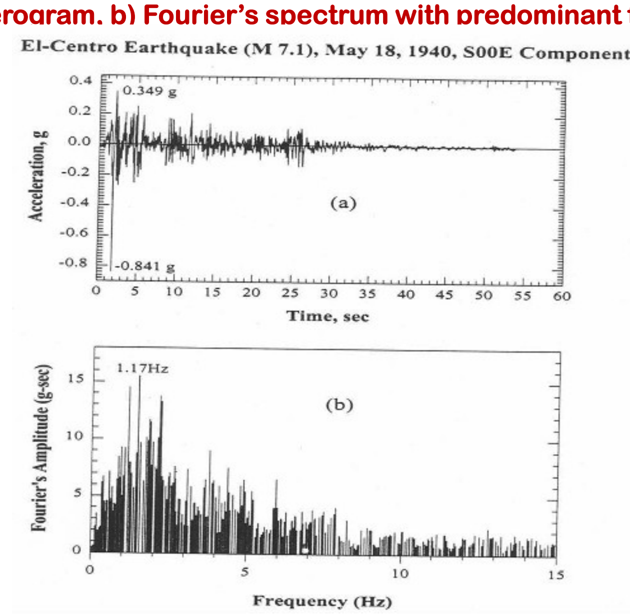
El – Centro earthquake of May18, 1940, SE component (M7.1): a) accelerogram, b) Fourier's spectrum with predominant frequency.



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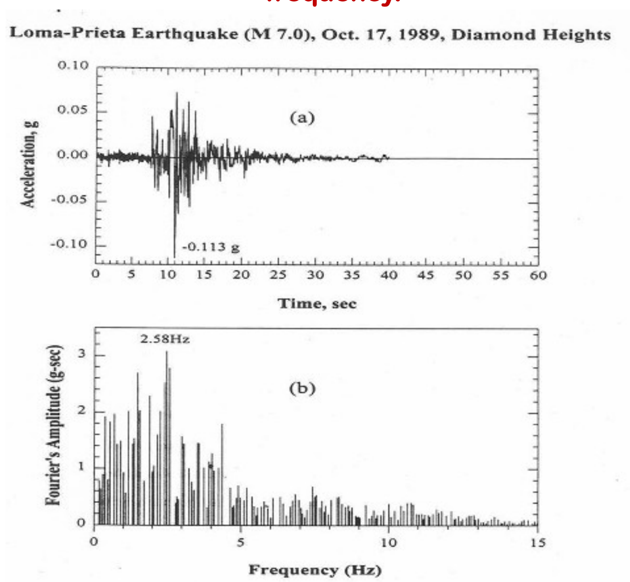
Loma – Prieta Earthquake (M 7.0), Oct. 17, 1989, Diamond Heights (M7.0): a) accelerogram, b) Fourier's spectrum with predominant frequency.



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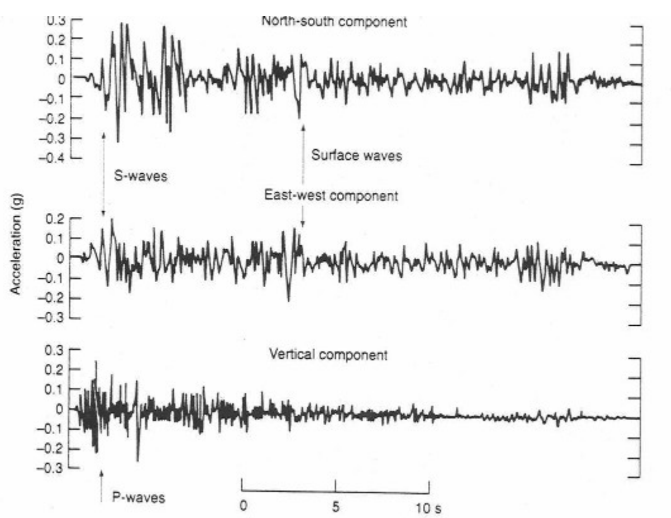
Seismograph or accelerogram record produced by seismograph



(b) Accelerogram of the 1940 Imperial Valley, California, earthquake recorded at El Centro, California.
(*Source:* W.W. Hays, 1980, *Procedures for Estimating Earthquake Ground Motions*, U.S. Geological Survey Professional Paper 1114)

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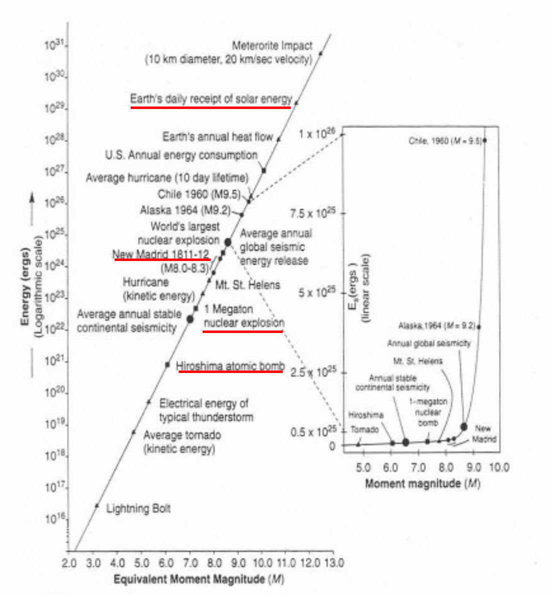
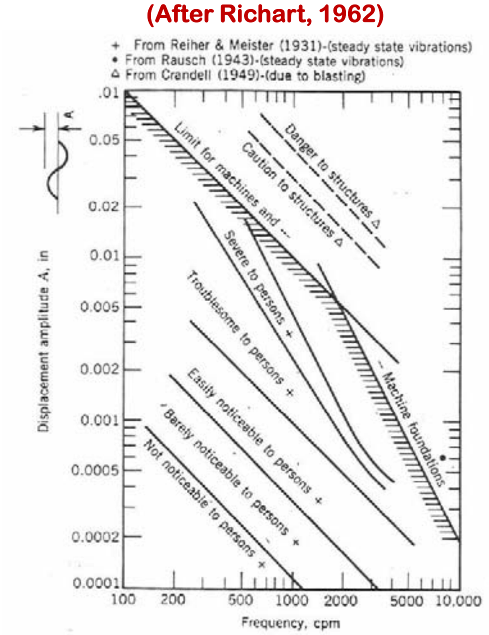
Relative energy of various natural and human – made phenomena.

(After Johnston. 1990.)



Limiting amplitudes of vibrations for a particular frequency.

(After Richart, 1962)



Criteria for vibrations of rotating machinery.

Explanations of classes:

AA Dangerous. Shut it down now to avoid danger.

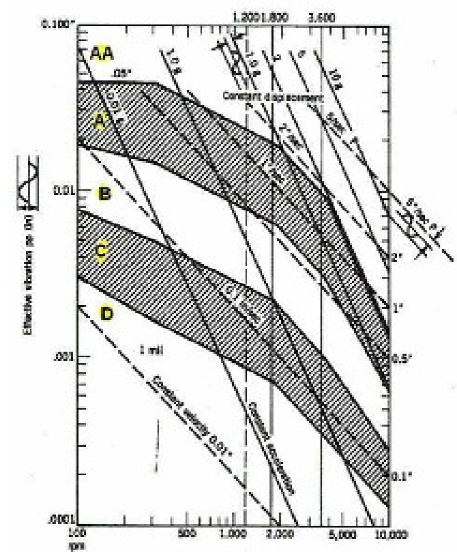
A Failure is near. Correct within two days to avoid breakdown.

B Faulty. Correct it within 10 days to save maintenance dollars.

C Minor Faults. Correction wastes dollars

D No faults. Typical new equipment.

This is a guide to aid judgment, not to replace it. Use common sense. Use with care. Take account of all local circumstances. Consider: safety, labor costs, downtime costs. (After Blake, 1964.)



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NEHRP Coefficients Aa and Av

Map Area from Map 1 (for A_a) or Map 2 (for A_v)	Value of A_a and A_v
7	0.40
6	0.30
5	0.20
4	0.15
3	0.10
2	0.05
1	< 0.05[a]

[a] For equations or expressions incorporating the terms A_a or A_v, a value of 0.05 shall be used.

For equations or expressions incorporating the terms Aa or Av a value of 0.05 shall be used.

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Maps of NEHRP seismic loading zones:

(a) map 1 for Aa

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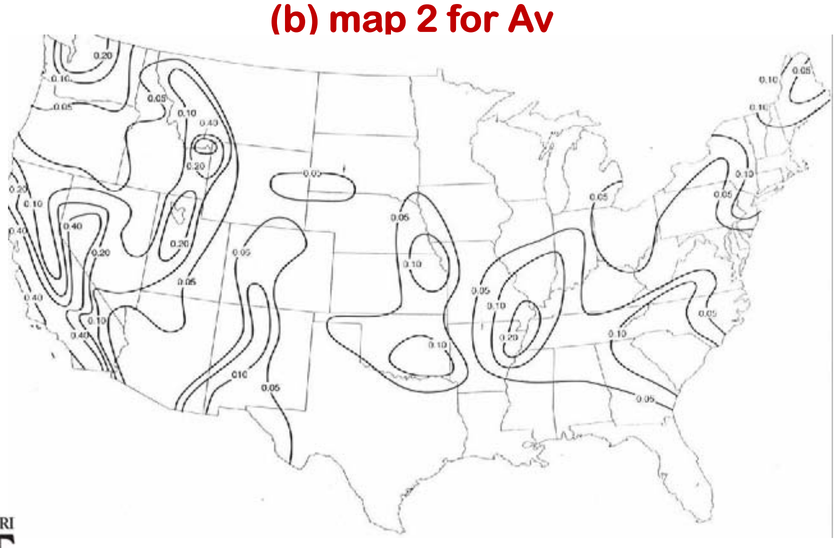
Maps of NEHRP seismic loading zones:

(b) map 2 for Av



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2. WAVE PROPAGATION

- P – WAVES
- S – WAVES
- R – WAVES
- BODY AND SURFACE WAVES

WAVE ISOLATION

- ACTIVE, PASSIVE

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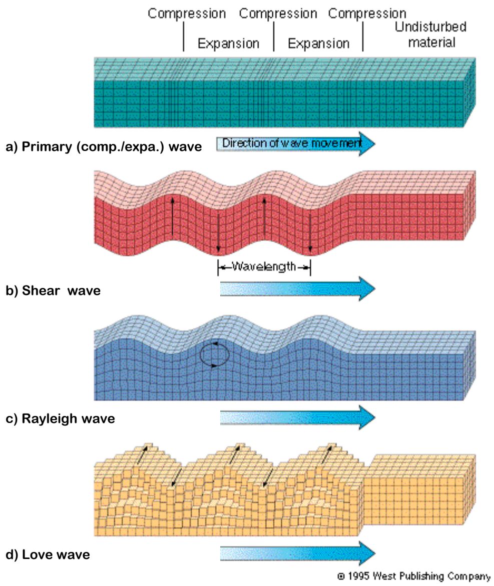

a) Direction of wave travel and particle motion is the same

b) Direction of particle motion is at right angles to that of wave travel

c) R-wave has both up and down motion in relation to direction of wave travel

d) L-wave forms a horizontal circle or ellipse moving in the direction of propagation

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TYPES OF WAVES

1. COMPRESSION (P) – WAVES

Direction of wave travel and particle motions are the same

$$V_P = \sqrt{\frac{E(1-v)}{\rho(1+v)(1-2v)}}$$

2. SHEAR (S)-WAVES

Direction of wave travel and particle motions are perpendicular to each other

$$V_s = \sqrt{\frac{G}{\rho}}$$

3. RAYLEIGH (R) - WAVE

Direction of particle motion is in 2-perpendicular directions to the direction of wave travel

$V_p > V_s$
$V_s \sim V_R$

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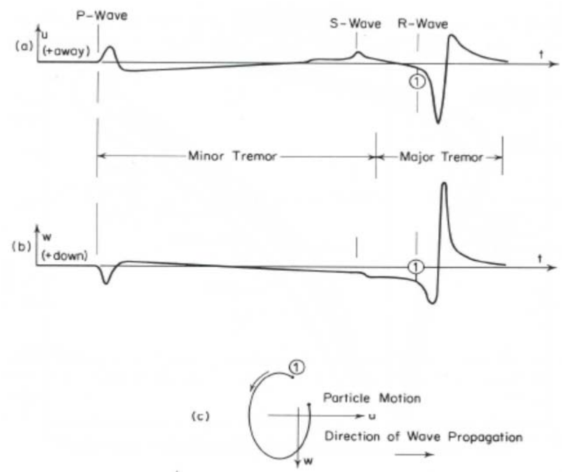
Wave system at a point from surface point source in ideal medium.



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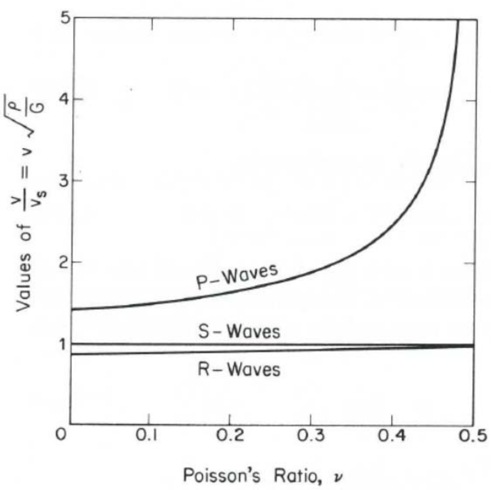
Variation of Rayleigh wave and body wave propagation velocities with Poisson's ratio.



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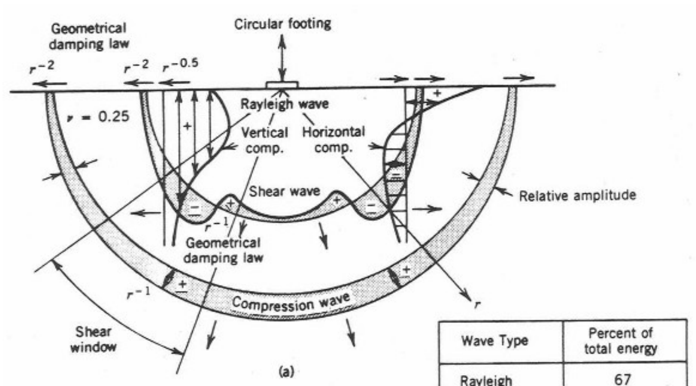
RADIATION DAMPING WAVE PROPAGATION IN AN ELASTIC MEDIUM

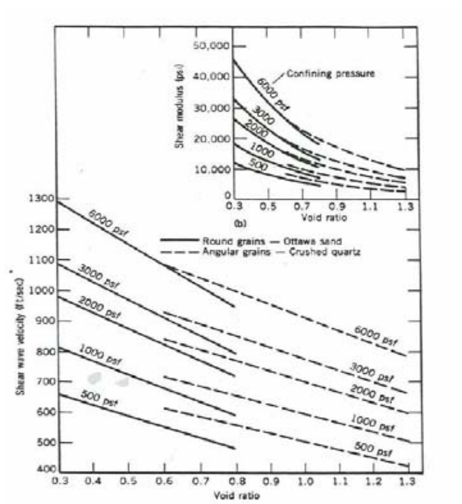


Wave Type	Percent of total energy
Rayleigh	67
Shear	26
Compression	7

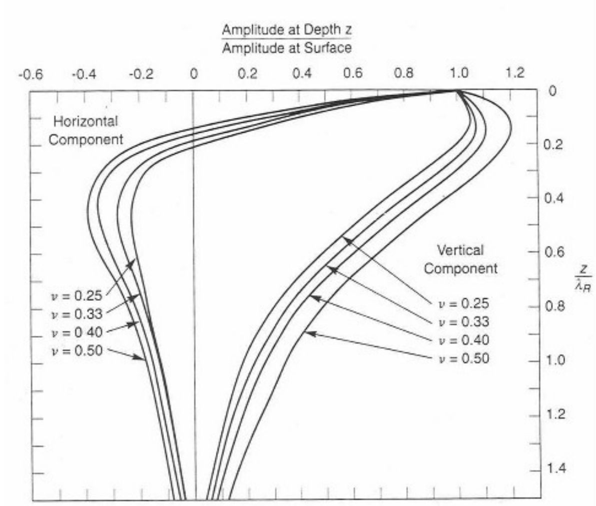
(b)

Distribution of displacement waves from a circular footing on a homogenous, isotropic, elastic half space (Woods, 1968)

Variation of shear wave velocity and shear modulus with void ratio and confining pressure for dry round and angular-grained sands.



Horizontal and vertical motion of Rayleigh waves. A negative amplitude ratio indicates that the displacement is in the opposite direction of the surface displacement.



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P and S-wave Velocities

	E, MN/m^2	ν	V_P, m/s	V_S, m/s
Moist clay	—	0.50	1500	150
Loess at natural moisture	100–130	0.44	800	260
Dense sand and gravel	—	—	480	250
Fine grained sand	85	0.30-0.35	300	110
Medium grained sand	83	0.30-0.35	550	160
Medium sized gravel	—	—	750	180
Rubber	1.97	0.50	43	27
Glass	55000	0.25	5300	3350
Copper	105000	0.34	3670	2250
Aluminum	69000	0.34	5030	3100
Steel	210000	0.29	5000	3220

P-wave Velocities in Rocks

Type of rock	*Velocity V_P, km/s*
Sand, gravel, silt	0.5–2.0
Shale, sandstone	1.5–4.5
Limestone	3.0–5.2
Dolomite	4.8–6.0
Granite	4.0–5.5
Basalt, gabbro	4.8–6.0

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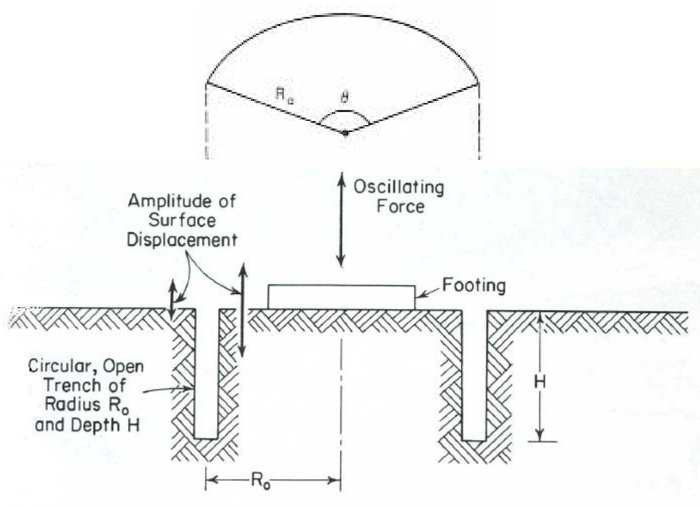
Schematic of vibration isolation using a circular trench surrounding the source of vibrations-active isolation.



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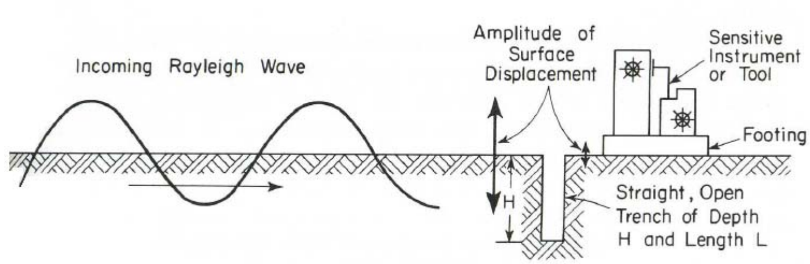
Schematic of vibration isolation using a straight trench – passive isolation.



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3.DAMAGE DURING EARTHQUAKE

- DAMAGE DUE TO LIQUEFACTION
- DAMAGE TO PILES
- MEXICO EARTHQUAKE
- LIFE LINES
- SURFACE FAULTING
- DAMAGE TO DAMS

DAMAGE DUE TO LIQUEFACTION

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SAND BOILS

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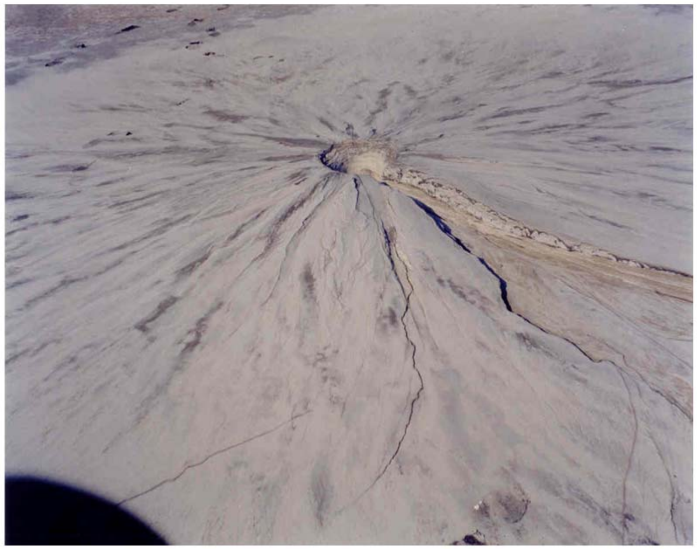
SAND BOILS



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Tilting of Buildings in Niigata (Japan) 1964

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Tilting of about 15 degrees

Another Tilted Building

A Septic Tank Moves Above Ground in Niigata

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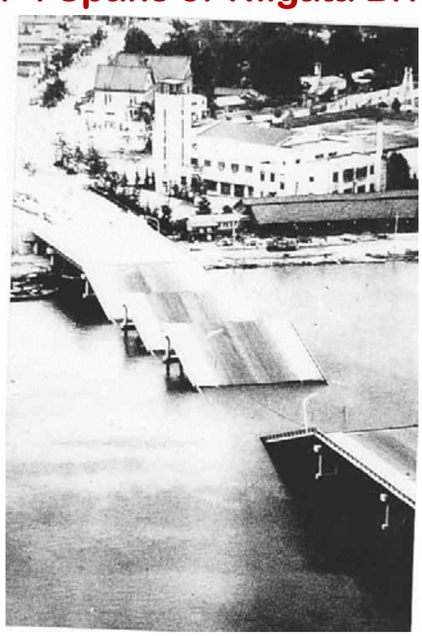
Failure of 4 Spans of Niigata Bridge 1964



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Tilt of Building in Guatemala EQ 1976

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Liquefaction During 2010 Haiti Earthquake

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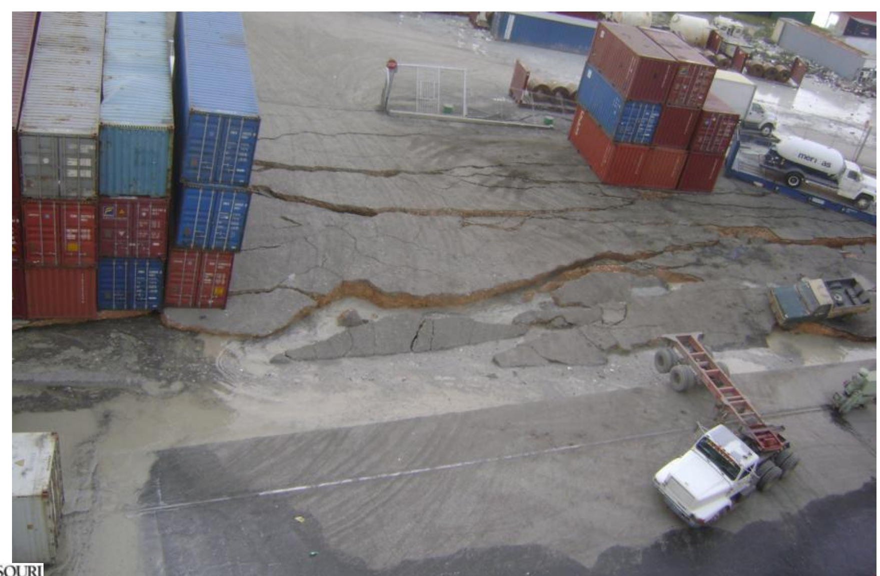
Effects on buildings (e.g., Kamisu City)

GEER 2011 (photo: Boulange

Liquefaction During 2010 Haiti Earthquake



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Liquefaction During 2010 Haiti Earthquake

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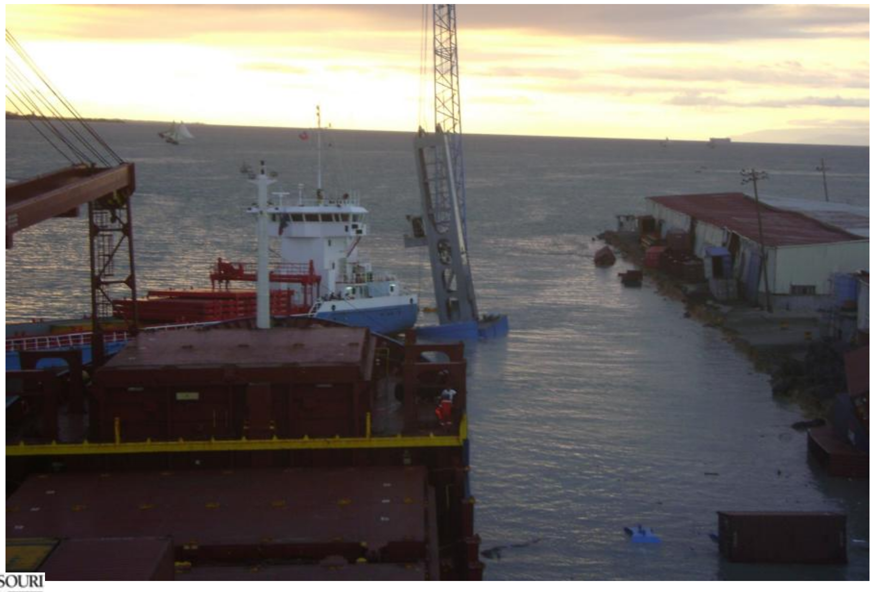
Liquefaction During 2010 Haiti Earthquake



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Liquefaction During 2010 Haiti Earthquake

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Mexicali 7.2 Earthquake on Rio Hardy, Mexico: The small river community is located approximately 40 miles southeast of Mexicali and estimated less than 5 miles from the epicenter. (2010)

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Mexicali 7.2 Earthquake on Rio Hardy, Mexico: The small river community is located approximately 40 miles southeast of Mexicali and estimated less than 5 miles from the epicenter. (2010)

57

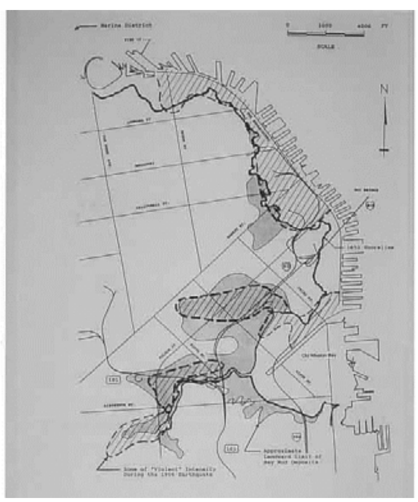

Fig. 7   Map of Eastern San Francisco Showing the Region Most Intensively Damaged During the 1906 Earthquake (Before the Post-Earthquake Fire), and the Historic Coastline and Marshes of 1852

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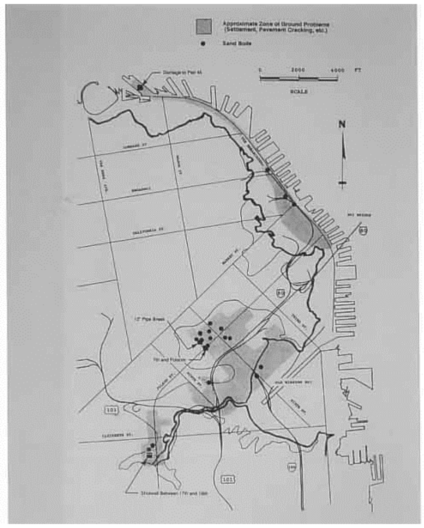

Fig. 8 Apparent Extent of Soil Liquefaction in San Francisco's Embarcadero and Old Mission Bay Regions on October 17, 1989

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CONCLUSION:

Liquefaction can occur at the same site again as in San Francisco 1906 and 1989

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Liquefaction in Loma Prieta Earthquake 1989

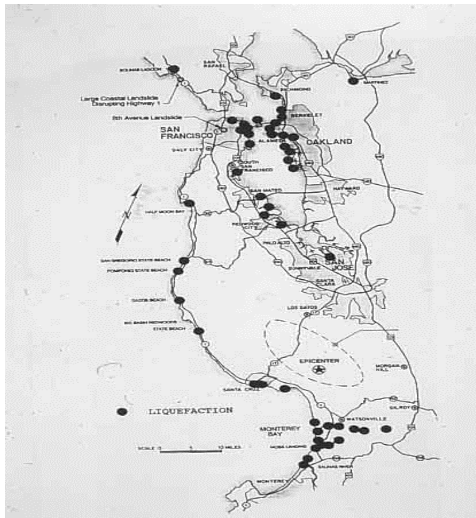

Fig. 1 Map of Affected Region Showing Sites of Soil Liquefaction

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Tsunami (maximum reported run up height of 38 m)

Photo by Reuters (framework.latimes.com)

Photo by Kyodo News (framework.latimes.com)

107,000 partial collapse/collapse & 230,000 damaged homes ***(Police – May 1, 2011).***

Photo by Kyodo Times (framework.latimes.com)

15,421 dead, 5,367 injured, & 7,937 missing ***(Police – June 5, 2011)***

Photo by STR/EPA (framework.latimes.com)

92% of victims drowned; 65% were >60 yrs old ***(Yomiuri Shinbun 4/19/11; Courtesy L. Johnson)***

Photo by Nicholas Kamm / APP / Getty Images

Response hindered by damages to roads, railways, airports, and port; e.g., first relief flights from Sendai airport were on March 17th.

AP Photo by Kyodo News

Loss of 561 km² (138,000 acres) along coast ***(Geospatial Info. Authority of Japan; L. Johnson)***

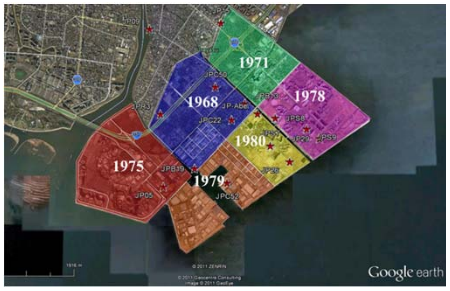
Photo by Kyodo Times (framework.latimes.com)

25 mil tons of debris will take 3 years to clean up *(Japan Times 4/2011)*

Photo by Adrees Latif/Reuters

Fukushima Dai-ichi nuclear power plant *Photo: DigitalGlobal, via Agence France-Presse – Getty Images*

Liquefaction in loose reclaimed land (a known pervasive hazard)



(photo Ishihara et al. 2012) *(photos by Urayasu City; Ishihara et al. 2012)*

DAMAGE TO DAMS

Naruse river - Levee & approach road at km 30

GEER 2011 (photo: Les Harder)

GEER 2011 (photo: L. F. Harder)

Fujinuma Auxiliary Dam – Upstream Slide


GEER 2011 (photo: Les Harder)

DAMAGE TO PILES

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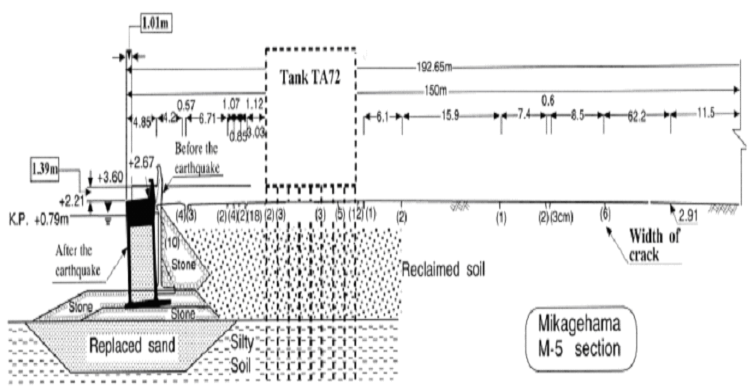
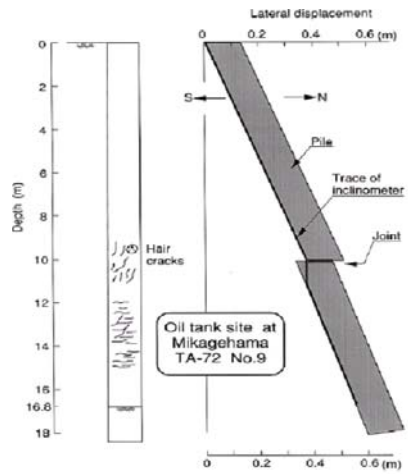
Damage to pile by 2m of lateral ground displacement during 1964 Niigata earthquake (Yosuda et al.1999)

Detailed profiles of the quay wall movement and ground distortion in the backfills at Section M-5 (Ishihara and Cubrinovski, 2004)



Lateral displacement and observed cracks on the inside wall of Pile No. 9 Kobe 1995 EQ (Ishihara and Cubrinovski, 2004)



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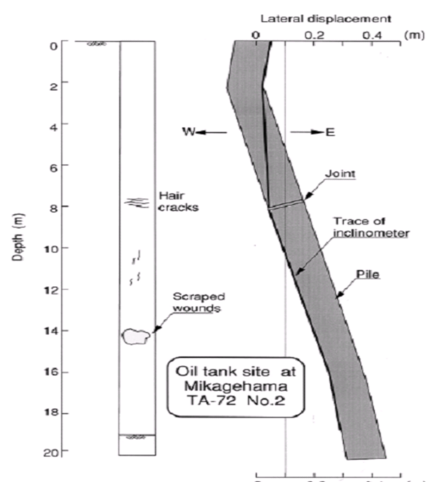
Lateral displacement and observed cracks on the inside wall of Pile No. 2 Kobe 1995 EQ (Ishihara and Cubrinovski, 2004)



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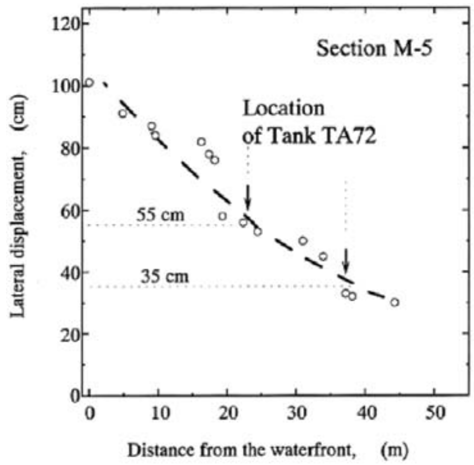
Lateral ground displacement versus distance from the waterfront along Section M-5, Kobe 1995 EQ (Ishihara and Cubrinovski, 2004)



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MEXICO EARTHQUAKE

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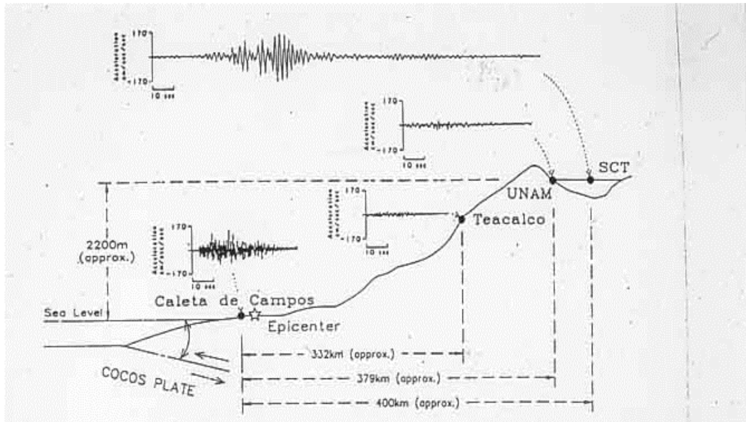

Figure 2- Schematic section showing relative locations of the epicentral station at Caleta de Campos, Teacalco station (closest to Mexico City), and Mexico City Stations, UNAM (hills zone) and SCT (lake zone). The seismograms are east-west components of 19 September 1985 acceleration time-histories (all plotted to the same scale) recorded at retrospective stations and demonstrate the attenuation of motions with distance from the coast as well as amplification of motions at the lakebed of Mexico City.

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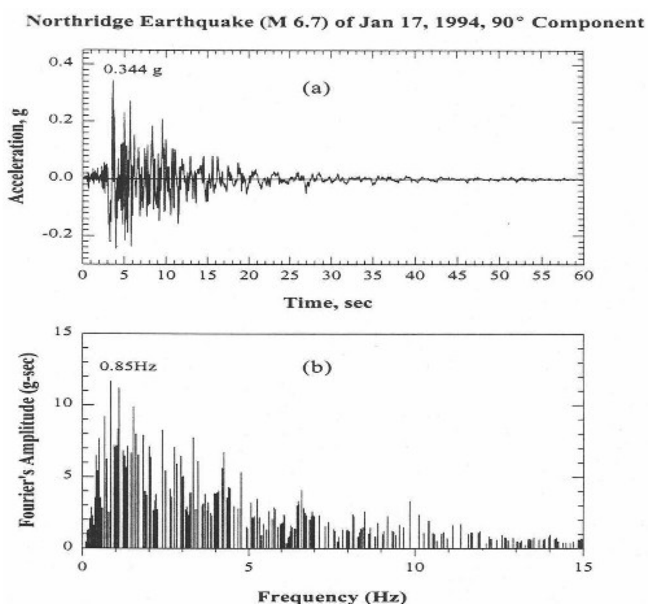
Northridge earthquake of Jan 17, 1994, 90 degree component (M6.7): a) accelerogram, b) Fourier's spectrum with predominant frequency.



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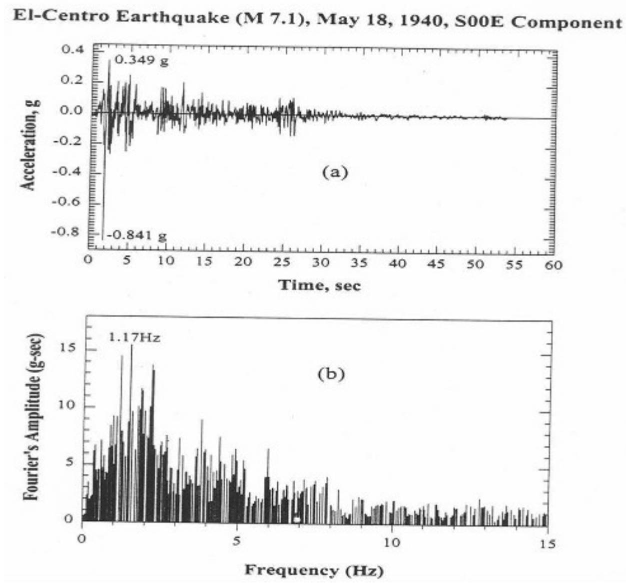
El – Centro earthquake of May18, 1940, SE component (M7.1):
a) accelerogram, b) Fourier’s spectrum with predominant frequency.



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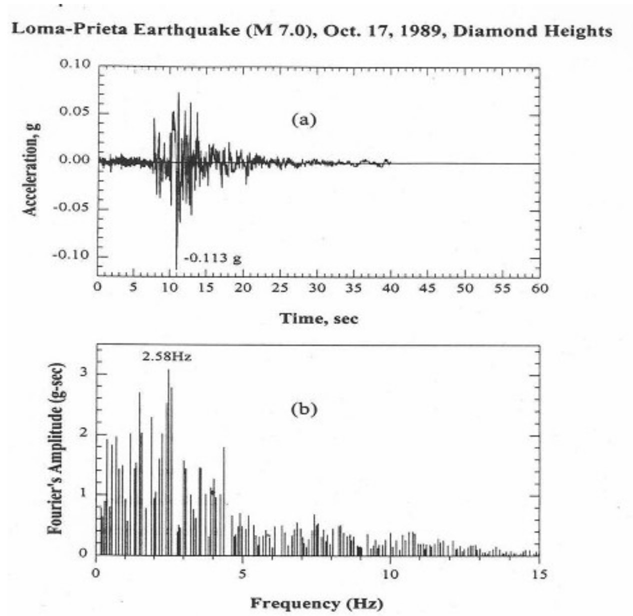
Loma – Prieta Earthquake (M 7.0), Oct. 17, 1989, Diamond Heights (M7.0):
a) accelerogram, b) Fourier’s spectrum with predominant frequency.



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Front – 8 – Storied Building Collapsed
Back – 15 Storied Building DID NOT
(Mexico 1985)

CONCLUSION

Double amplification had been observed in a significant manner first time in an earthquake:

a) From rock to soft soil surface
b) From soil surface to top of the building
c) Dominant frequency of ground motion controls damage to buildings in a significant way

PREFAB Construction
(Mexico 1985)

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SURFACE FAULTING

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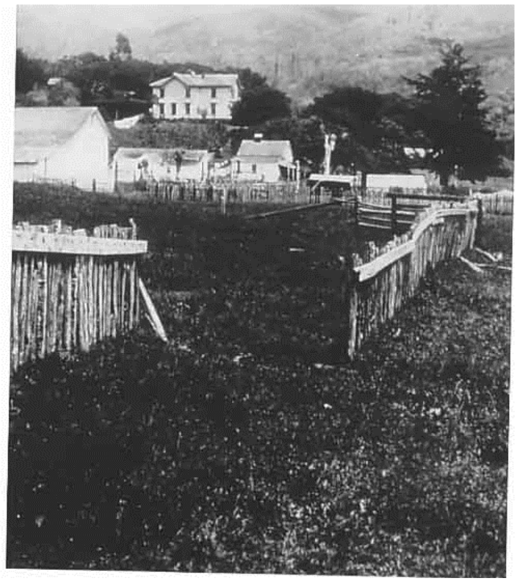
Orange County (CA)
Break in (about 8 feet) Wall about 8'



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Soft Story Effect

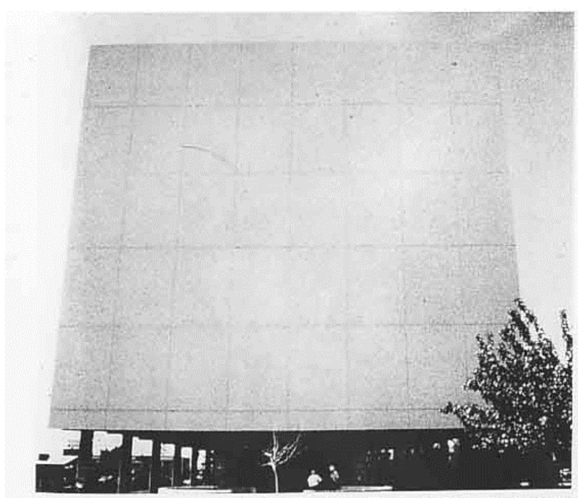

Figure 3-2. Failed columns in the Imperial County Services Building, 1979. (Photo: Helmut Krawinkler).

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SOIL DYNAMICS AND MODELING

1. SOURCE OF DYNAMIC LOADING
2. WAVE PROPAGATION
3. DAMAGE DURING EARTHQUAKE
4. IDEALIZATION OF SOILS AND STRUCTURES FOR ANALYSIS
5. VIBRATION ANALYSIS

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4. IDEALIZATION OF SOIL STRUCTURES FOR ANALYSIS

- DISCRETE SYSTEMS
- DISTRIBUTED MASSES SYSTEMS

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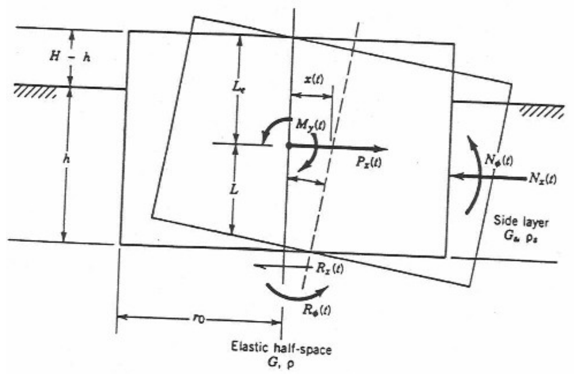
DISCRETE SYSTEM



Mathematical model of rigid block embedded in elastic half – space with soil side layer in coupled motion

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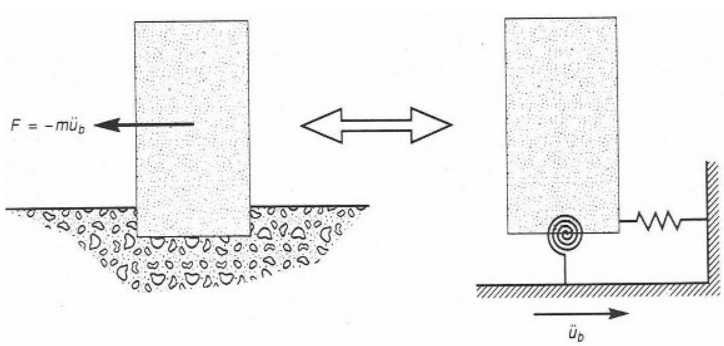
Equivalent formulations of inertial interaction analysis for structures with rigid foundation



Inertia forces applied to each element

Foundation motion applied through frequency – dependent springs and dashpots (not shown)

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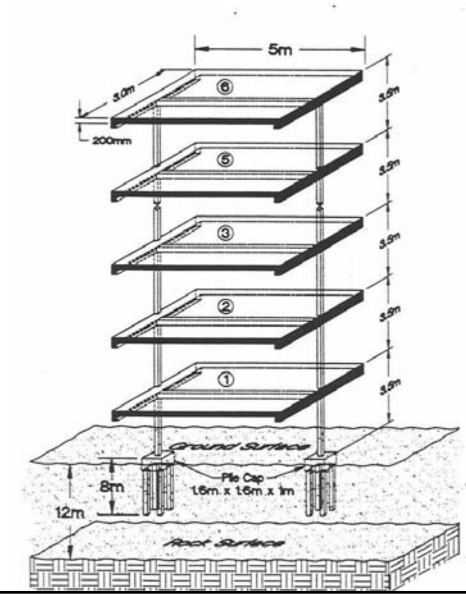
Typical n – story frame



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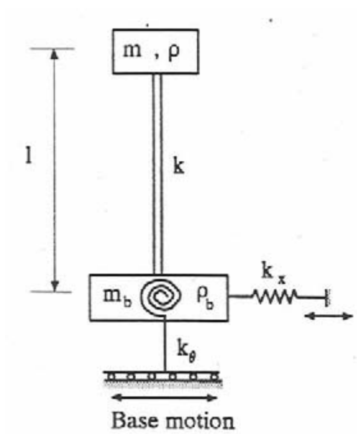
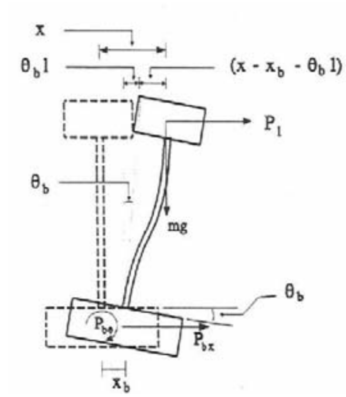


Mathematical model for a single story frame on flexible foundation



Deflected shape of a single story frame

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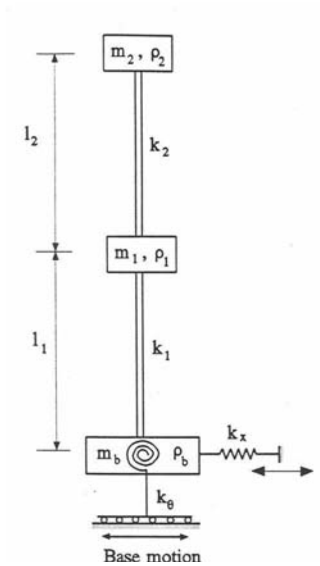

Mathematical model for two story frame with flexible foundation

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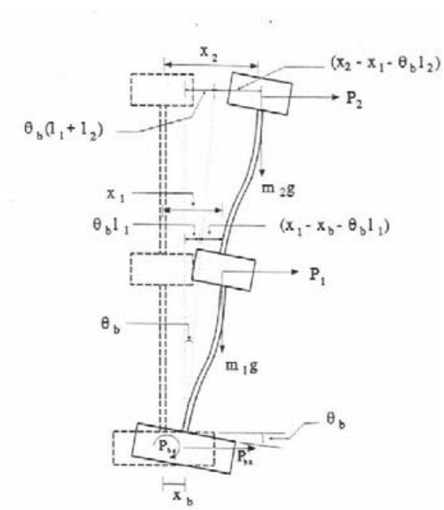
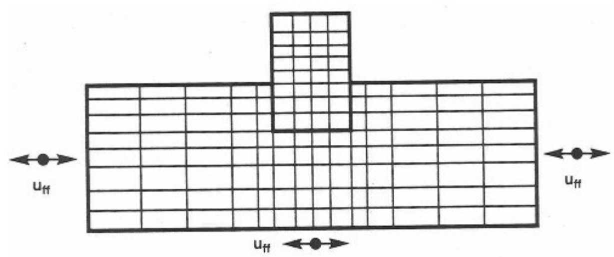


Deflected shape of a story frame

100



Direct method of soil – structure interaction analysis. Entire problem is modeled and response to free – field motion applied at boundaries is determined in a single step

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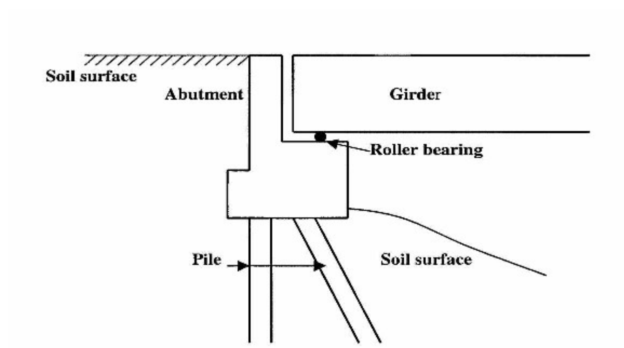

TYPICAL HIGHWAY BRIDGE ABUTMENT SUPPORTED ON PILES

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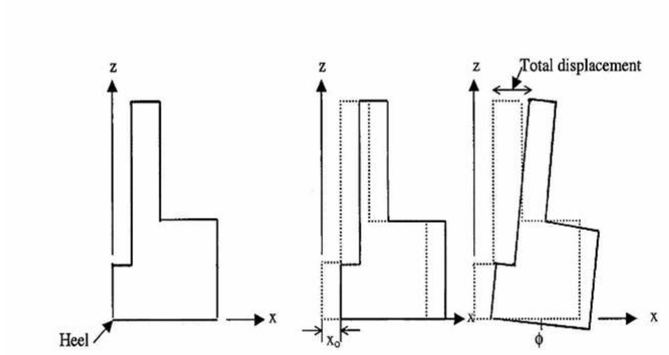
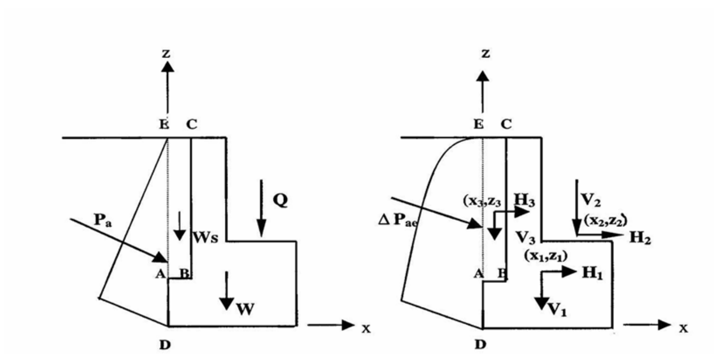


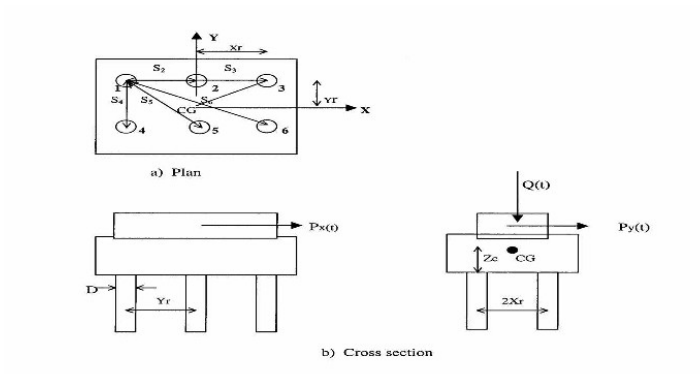
a. Initial Condition **b. Sliding** **c. Sliding and Rotation**

TRANSLATION AND ROTATION MOVEMENT OF ABUTMENT



a) Static forces **b) Dynamic force increments**

FORCES ACTING ON THE BRIDGE ABUTMENT



Plan and Cross Section of Pile Group

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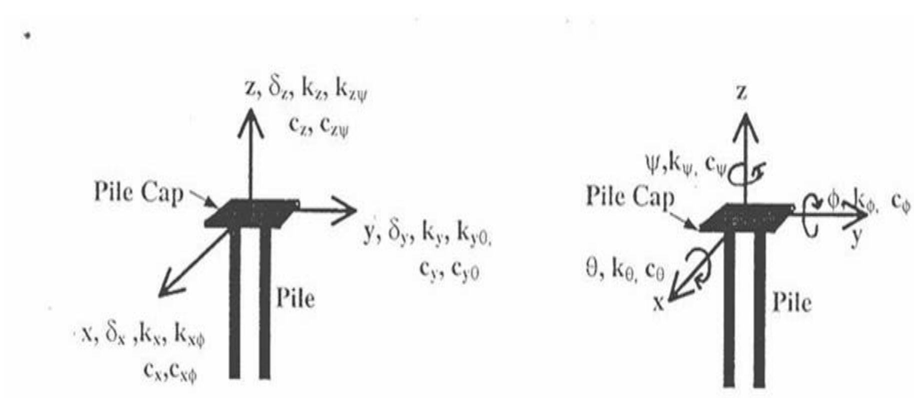
SIGN CONVENTIONS



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EIGHT SPRING CONSTANTS

k_x, k_y, k_z TRANSLATION

k_θ, k_ϕ, k_ψ ROTATION

$k_{x\phi}, k_{y\theta}$ CROSS-COUPLING

EIGHT DAMPING CONSTANTS

c_x, c_y, c_z TRANSLATION

c_θ, c_ϕ, c_ψ ROTATION

$c_{x\theta}, c_{y\theta}$ CROSS-COUPLING

SOIL DYNAMICS AND MODELING

1. SOURCE OF DYNAMIC LOADING
2. WAVE PROPAGATION
3. DAMAGE DURING EARTHQUAKE
4. IDEALIZATION OF SOILS AND STRUCTURES FOR ANALYSIS
5. VIBRATION ANALYSIS

5. VIBRATION ANALYSIS

- SPRING – MASS – DASHPOT SYSTEM
- NATURAL FREQUENCY
- DAMPING: NATURE OF DAMPING
  - Viscous Damping
  - Friction Damping
  - Radiation Damping
  - Total Damping
- SINGLE DEGREES OF FREEDOM SYSTEM (SDOF)
- TWO DEGREES OF FREEDOM SYSTEM (2DOF)
- MULTI DEGREES OF FREEDOM SYSTEM (MDOF)
- CONCLUDING REMARKS

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IMPORTANT DEFINITIONS

- NATURAL FREQUENCY
- DEGREES OF FREEDOM
- DAMPING
- CRITICAL DAMPING
- ORDER OF DAMPING IN MATERIALS
- ORDER OF DAMPING IN STRUCTURES
  - LINEAR DAMPING
  - NON LINEAR DAMPING

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A. THEORY OF VIBRATIONS

Simple theoretical concepts of harmonic vibrations

B. DEFINITIONS

PERIOD: If motion repeats itself in equal intervals of time, it is called a ***periodic motion*** and the time elapsed in repeating the motion once is called its ***period***

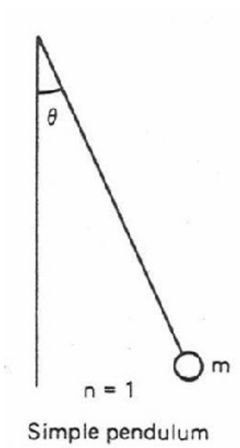
CYCLE: Motion completed during a period is referred to as a ***cycle***

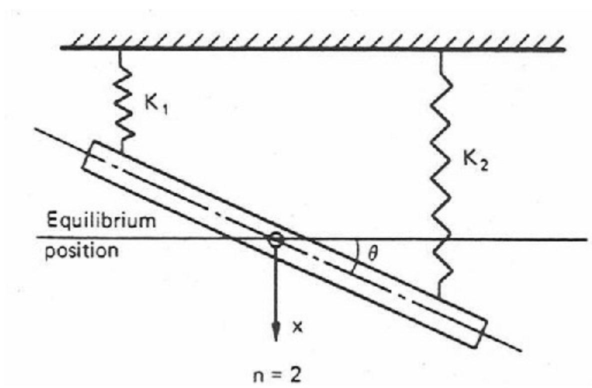
FREQUENCY: The number of cycles of motion in a unit of time is called the ***frequency*** of vibration

NATURAL FREQUENCY: If an elastic system vibrates under the action of forces in the system and in the absence of any externally applied force, the frequency with which it vibrates is its ***natural frequency***

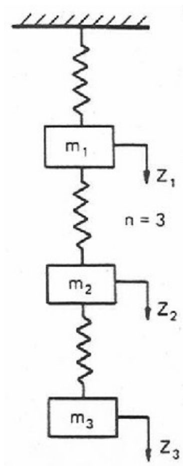
FORCED VIBRATIONS: Vibrations that occur under the excitation of external forces are termed ***forced vibrations***. Forced vibrations occur at a frequency of the exciting force. The frequency of excitation is independent of the natural frequency of the system.

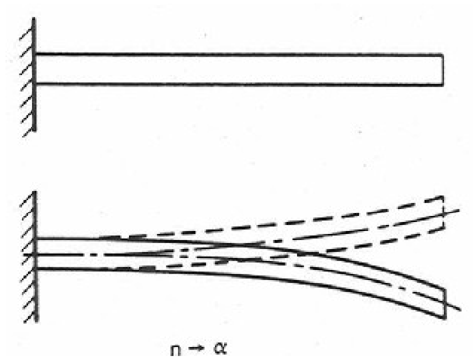
DEGREES OF FREEDOM: The number of independent coordinates necessary to describe the motion of a system specifies the ***degrees of freedom of the system***. A system may in general have several degrees of freedom; such a system is called a ***multidegree freedom*** system.





One degree of freedom (n=1) **Two degrees of freedom (n=2)**





Three degrees of freedom (n=3) **Infinite degrees of freedom (n= ∞)**

DAMPING

Damping force (F_d) is resistance to motion of an oscillating system

1. **Viscous damping (C)**

$$F_d \propto \text{Velocity}$$

i.e. $F_d = C.X$

where, C = coefficient of viscous damping

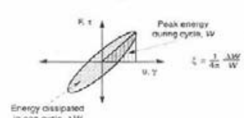
2. **Friction damping (C_f)**

$$F_d \propto \text{weight}$$

i.e $F_d = W.(C_f) = W.\mu$

where, μ = coefficient of friction

3. **Material damping (C_m)**



4. **Radiation or geometrical damping (C_r)**

Energy dissipated in an ELASTIC half space

5. **Total damping**

$$C_{total} = C_{material} + C_{radiation}$$

6. **Critical damping C_c**

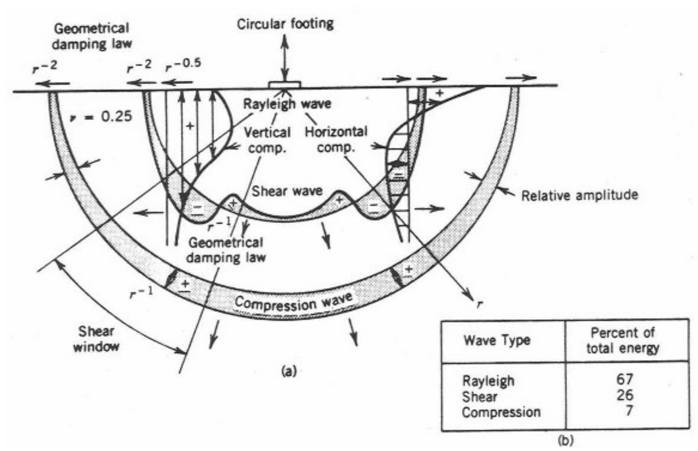
A physical condition when a system does not oscillate if displaced from equilibrium and returns to original position in minimum time(t_{min}).

However, if $t > t_{min}$ $C > C_c$ The system still non-oscillatory.

7. **Damping factor**

$$\xi = C/C_c$$

RADIATION DAMPING WAVE PROPAGATION IN AN ELASTIC MEDIUM

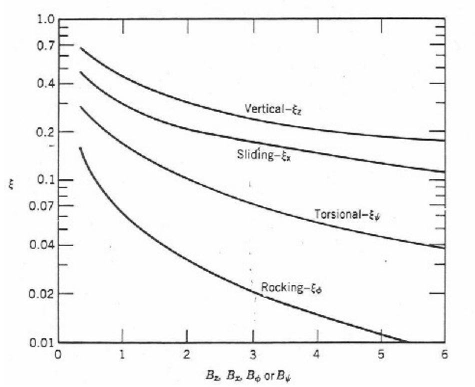


Distribution of displacement waves from a circular footing on a homogenous, isotropic, elastic half space (Woods, 1968)

Mass ration B, damping factor ξ, and spring constant k for rigid circular footing on the semi – (static k'$_s$) infinite elastic half space

Mode of vibration	Mass (or inertia) ratio	Damping factor	Spring constant
1	2	3	4
Vertical	$B_z = \frac{(1-\nu)}{4}\frac{m}{\rho r_0^3}$	$\xi_z = \frac{0.425}{\sqrt{B_z}}$	$k_z = \frac{4Gr_0}{1-\nu}$
Sliding	$B_x = \frac{(7-8\nu)}{32(1-\nu)}\frac{m}{\rho r_0^3}$	$\xi_x = \frac{0.2875}{\sqrt{B_x}}$	$k_x = \frac{32(1-\nu)}{7-8\nu}Gr_0$
Rocking	$B_\phi = \frac{3(1-\nu)}{8}\cdot\frac{M_{m0}}{\rho r_0^5}$	$\xi_\phi = \frac{0.15}{(1+B_\phi)\sqrt{B_\phi}}$	$k_\phi = \frac{8Gr_0^3}{3(1-\nu)}$
Torsional	$B_\psi = \frac{J_z}{\rho r_0^5}$	$\xi_\psi = \frac{0.5}{1+2B_\psi}$	$k_\psi = \frac{16}{3}\cdot Gr_0^3$

RADIATION DAMPING IN SOILS



Equivalent damping ratio for oscillation of rigid circular footing on the elastic half – space

DEGREES OF FREEDOM OF A BLOCK FOUNDATION

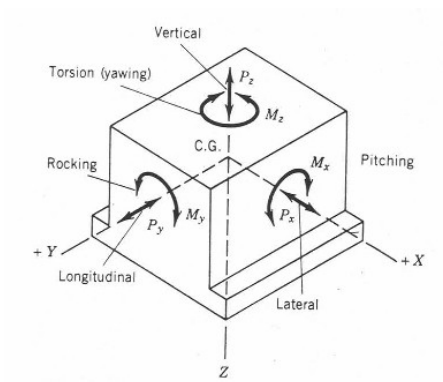
SIX DEGREES OF FREEDOM:

1. Translation along Z axis
2. Translation along X axis
3. Translation along Y axis
4. Rotation along Z axis
5. Rotation along X axis
6. Rotation along Y axis

Coupled Motion:

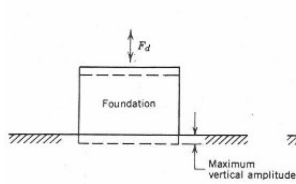
a. 2 and 6
b. 3 and 5

Rigid Block / Elastic Soil

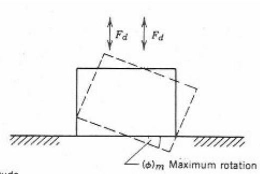


Modes of vibration of a rigid block foundation

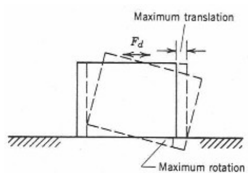
TYPES OF UNBALANCED LOADS OF MACHINES ON FOUNDATIONS



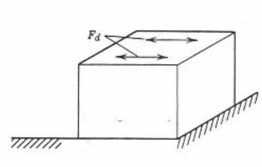
Pure vertical translation



Pure rocking



Simultaneous horizontal sliding and rocking



Pure torsional oscillations

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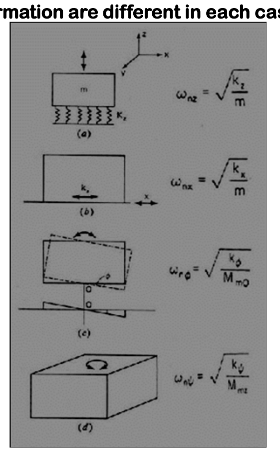
DEFINITION OF SOIL SPRING STIFFNESS

a. Uniform compression
b. Uniform shear
c. Non-uniform compression
d. Non-uniform shear

Therefore, the soil constant characterizing the stress below the block and the corresponding elastic deformation are different in each case



a. Vertical Vibrations

b. Horizontal Translations

c. Rocking

d. Yawing

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METHODS OF ANALYSIS

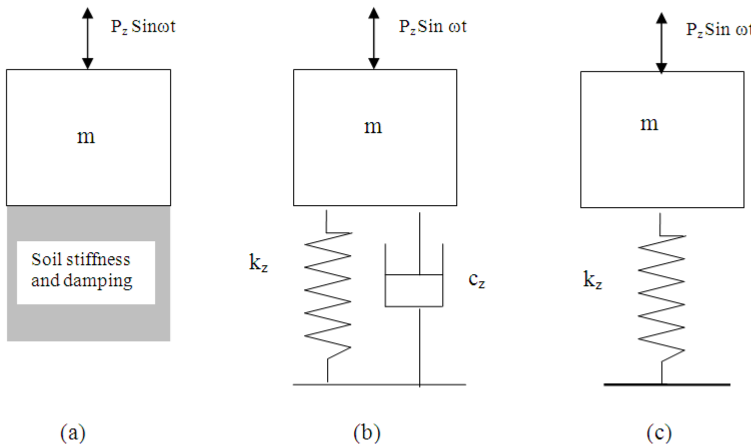

Figure 5. ***Vertical Vibrations of a Machine Foundation (a) Actual case, (b) Equivalent model with damping (c) Model without damping***

Elastic-half –space -analogs

Surface Foundations

Vertical vibrations: The problem of vertical vibrations is idealized as a single degree freedom system with damping as shown in Fig. 13.15b. Hsieh (1962) and Lysmer and Richart (1966) have provided a solution .The equation of vibration is:

$$m\ddot{z} + \frac{3.4r_o^2}{(1-v)}\sqrt{\rho G}\dot{z} + \frac{4Gr_o}{(1-v)}s = P_z \sin(\omega t) \qquad 1$$

Where r_o = radius of the foundation (For non-circular foundations, appropriate equivalent radius may be used, see Eqs. 40-42).

The equivalent spring for vertical vibrations is given by

$$k_z = \frac{4Gr_o}{1-v} \qquad 2$$

And the damping c_z is given by

$$c_z = \frac{3.4r_o}{1-v}\sqrt{\rho G} \qquad 3$$

The damping constant for vertical vibrations ξ_z is given by

$$\xi_z = \frac{0.425}{\sqrt{B_z}} \qquad 4$$

In which B_z is known as the modified mass ratio, given by

$$B_z = \frac{1-v}{4} \cdot \frac{m}{\rho r_o^3} \qquad 5$$

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The undamped natural frequency of vertical vibrations may now be obtained using Eqs. 6 and 7.

$$\omega_{nz} = \sqrt{\frac{k_z}{m}} \qquad 6$$

$$f_{nz} = \frac{1}{2\pi}\sqrt{\frac{k_z}{m}} \qquad 7$$

In which ω_{nz} = the circular natural frequency (undamped) of the soil foundation system in vertical vibration (rad/sec) and f_{nz} = natural frequency of vertical vibrations (Hz).

The amplitude of vertical vibration is obtained as:

$$A_z = \frac{P_z}{k_z\sqrt{(1-r^2)^2 + (2\xi_z r)^2}} = \frac{P_z}{k_z\left\{\left[1-(\frac{\omega}{\omega_{nz}})^2\right]^2 + (\frac{2\xi_z\omega}{\omega_{nz}})^2\right\}^{1/2}} \qquad 8$$

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Sliding vibrations

The equation of the analog for sliding is (Fig. 6)

$$m\ddot{x} + c_x + k_x x = P_z sin(\omega t) \quad 9$$

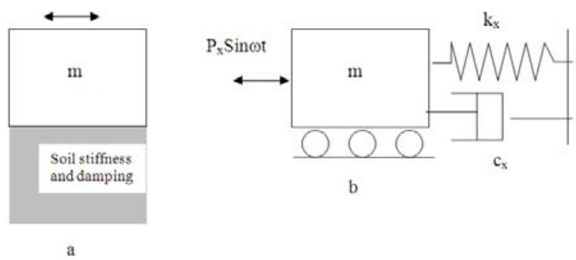

Figure 6. *Sliding Vibrations of a Rigid Block (a) Actual case (b) Equivalent model*

Hall (1967) defined the modified mass ratio for sliding as:

$$B_x = \frac{7-8v}{32(1-v)}\frac{m}{pr_o^3} \quad 10$$

where r_o = radius of the foundation .

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The expressions for the equivalent spring and damping factors are as follows:
The equivalent spring

$$k_x = \frac{32(1-v)}{7-8v} Gr_o \quad 11$$

And the equivalent damping

$$c_x = \frac{18.4(1-v)}{7-8v} r_o^2 \sqrt{\rho G} \quad 12$$

The damping ratio ξ_x is given by

$$\xi_x = \frac{c_x}{c_e} = \frac{0.2875}{\sqrt{B_x}} \quad 13$$

The undamped natural frequency of sliding vibration may be obtained as follows:

$$\omega_{nx} = \sqrt{\frac{k_x}{m}} \quad 14a$$

$$f_{nx} = \frac{1}{2\pi}\sqrt{\frac{k_x}{m}} \quad 14b$$

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In which ω_{nx} = the circular natural frequency (undamped) in sliding vibrations and f_{nx} = natural frequency of sliding vibrations (Hz).

The damped amplitude in sliding is obtained as:

$$A_x = \frac{P_x}{k_x\sqrt{\left(1-\left(\frac{\omega}{\omega_{nx}}\right)^2\right)^2+\left(2\xi_x\frac{\omega}{\omega_{nx}}\right)^2}} \qquad 15$$

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Rocking Vibrations: **A rigid block foundation undergoing rocking vibrations due to an exciting moment M_y sin ωt is shown in Fig. 7.**

Hall (1967) proposed an equivalent mass-spring-dashpot model that can be used to determine the natural frequency and amplitude of vibration of a rigid circular footing resting on an elastic half-space and undergoing rocking vibrations (Fig.7). The equivalent model is given in equation 16

$$M_{mo}\ddot{\varphi} + c_\varphi\dot{\varphi} + k_\varphi\varphi = M_y\sin(\omega t) \qquad 16$$

In which k_ϕ = spring constant for rocking, c_ϕ = damping constant and M_{mo} = mass moment of inertia of the foundation and machine about the axis of rotation through the base.

$$M_{mo} = M_m + mL^2 \qquad 17$$

Where M_m = mass moment of inertia of foundation and machine about an axis passing through the centroid of the system and parallel to the axis of rotation and L = the height of the centroid above the base.

The terms k_ϕ *and* c_ϕ can be obtained as follows:

$$k_\varphi = \frac{8Gr_o^3}{3(1-v)} \qquad 18$$

And

$$c_\varphi = \frac{0.8r_o^4\sqrt{G\rho}}{(1-v)(1+B_\varphi)} \qquad 19$$

in which $r_{0\phi}$ = radius.

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B_ϕ in Eq. 19 is known as the modified inertia ratio which obtained as follows:

$$B_\varphi = \frac{3(1-v)}{8}\frac{M_{mo}}{\rho r_o^5} \qquad 20$$

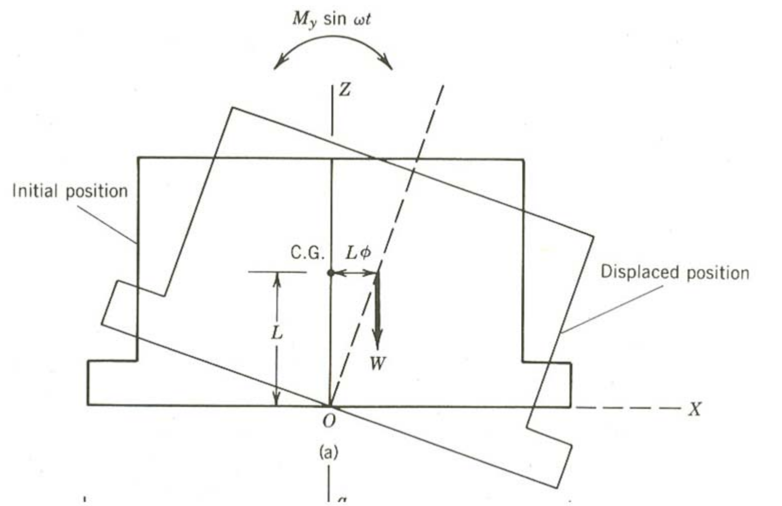

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Figure 7. Rocking vibrations of a rigid block under excitation due to an applied moment

The damping factor ξ_ϕ is given by

$$\xi_\varphi = \frac{c_\varphi}{c_\varphi} = \frac{0.15}{(1+B_\varphi)\sqrt{B_\varphi}} \qquad 21$$

The undamped natural frequency of rocking

$$\omega_{n\varphi} = \sqrt{\frac{k_\varphi}{M_{mo}}} rad/sec \qquad 22$$

Damped amplitude of rocking vibrations A_ϕ is given by Eq. 23

$$A_\varphi = \frac{M_y}{k_\varphi\sqrt{\left(1-\left(\frac{\omega}{\omega_{n\varphi}}\right)^2\right)^2 + \left(2\xi_\varphi\frac{\omega}{\omega_{n\varphi}}\right)^2}} \qquad 23$$

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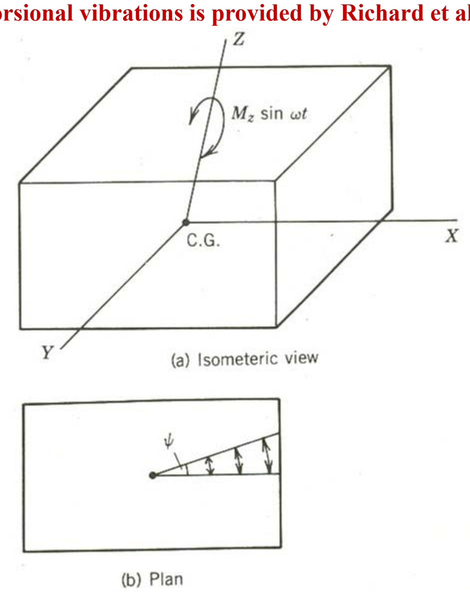
Torsional vibrations: **A block foundation undergoing torsional vibrations is shown in Fig.8. Non-uniform shearing resistance is mobilized during such vibrations. The analog solution for torsional vibrations is provided by Richard et al, (1970).**



<u>Figure 8:</u> ***Torsional vibrations of rigid block: (a) Block subjected to horizontal moment. (b) Development of nonuniform shear below the base***

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The equation of motion is

$$M_{mz}\ddot{\Psi} + C_\psi \dot{\Psi} + k_\psi \Psi = M_z e^{i\omega t} \qquad 24$$

In which M_{mz} = mass moment of inertia of the machine and foundation about the vertical axis of rotation (polar mass moment of inertia). The spring constant k_ψ and the damping constant c_ψ are given by (Richart and Whitman, 1967):

$$k_\psi = \frac{16}{3} G r_o^3 \qquad 25$$

$$c_\psi = \frac{1.6 r_o^4 \sqrt{G\rho}}{1 + B_\psi} \qquad 26$$

where $r_o\,(r_{o\psi})$ = equivalent radius..

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The undamped natural frequency $\omega_{n\psi}$ of the torsional vibrations is given by

$$\omega_{n\Psi} = \sqrt{\frac{k_\Psi}{M_{mz}}}\, rad/sec \qquad 27$$

The amplitude of vibration A_ψ is given by

$$A_\Psi = \frac{M_z}{k_\Psi \sqrt{\left(1 - \left(\frac{\omega}{\omega_{n\Psi}}\right)^2\right)^2 + \left(2\xi\Psi \frac{\omega}{\omega_{n\varphi}}\right)^2}} \qquad 28$$

In which the damping ratio ξ_ψ is given by

$$\xi_\Psi = \frac{0.5}{(1 + 2B_\Psi)} \qquad 29$$

The modified inertia ratio B_ψ is given by

$$B_\Psi = \frac{M_{mz}}{\rho r_o^5} \qquad 30$$

Combined rocking and sliding: **The problem of combined rocking and sliding is shown schematically in Fig. 9. The equations of motion are written as:**

$$m\ddot{x} + c_x\dot{x} + k_x x - Lc_x\dot{\varphi} - Lk_x\varphi = P_x e^{i\omega t} \qquad 31$$

$$M_m\ddot{\varphi} + (c_\varphi + L^2 C_x)\dot{\varphi} + (k_\varphi + L^2 k_x)\varphi - Lc_x\dot{x} - Lk_x x = M_y e^{i\omega t} \qquad 32$$

The undamped natural frequencies for this case can be obtained from Eq. 33.

$$\omega_n^4 - \left(\frac{\omega_{nx}^2 - \omega_{n\varphi}^2}{\gamma}\right)\omega_n^2 + \frac{\omega_{nx}^2 \cdot \omega_{n\varphi}^2}{\gamma} = 0 \qquad 33$$

In which

$$\gamma = \frac{M_m}{M_{mo}} \qquad 34$$

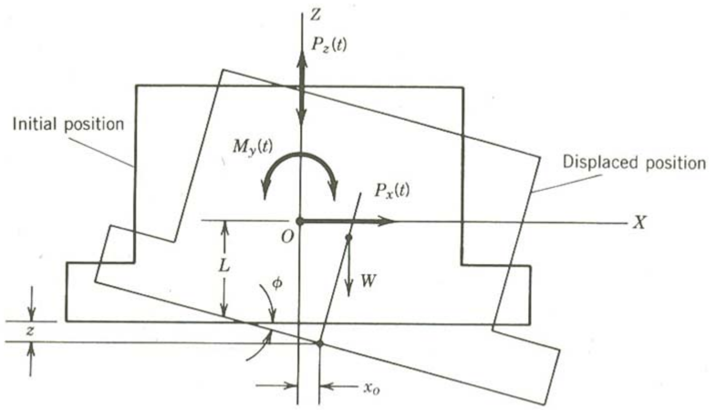

Figure 9. ***Block subjected to the action of simultaneous vertical $P_z(t)$, horizontal $P_x(t)$ forces and moment $M_y(t)$***

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The damping in rocking and sliding modes will be different. Prakash and Puri (1988) developed equations for determination of vibration amplitudes for this case. Damped amplitudes of rocking and sliding occasioned by an exciting moment M_y can be obtained as follows:

$$A_x = \frac{M_y}{M_m} \frac{[(\omega_{nx}^2)^2 + (2\xi_x \omega \omega_{nx})^2]^{1/2} L}{\Delta(\omega^2)} \qquad 356$$

$$A_\psi = \frac{M_y}{M_m} \frac{[(\omega_{nx}^2 - \omega^2)^2 + (2\xi_x \omega_{nx} \omega)^2]^{1/2}}{\Delta(\omega^2)} \qquad 36$$

The value of $\Delta(\omega^2)$ is obtained from Eq. 38

$$\Delta(\omega^2) = \left[\left(\omega^4 - \omega^2 \left\{ \frac{\omega_{n\varphi}^2 + \omega_{nx}^2}{\gamma} - \frac{4\xi_x \xi_\varphi \omega_{nx} \omega_{n\varphi}}{\gamma} \right\} + \frac{\omega_{nx}^2 \omega_{n\varphi}^2}{\gamma} \right)^2 + 4\left\{ \xi_x \frac{\omega_{nx}\omega}{\gamma} (\omega_{n\varphi}^2 - \omega^2) + \frac{\xi_\varphi \omega_{n\varphi} \omega}{\gamma} (\omega_{nx}^2 - \omega^2) \right\}^2 \right]^{1/2} \qquad 37$$

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Damped amplitudes of rocking and sliding occasioned by a horizontal force P_x are given by Eqs.38 and 39

$$A_x = \frac{P_x}{mM_m} \frac{\left[\left(1 - M_m\omega^2 + k_\varphi + L^2 k_x\right)^2 + 4\omega^2\left(\xi_\varphi\sqrt{k_\varphi M_{mo}} + L^2\xi_x\sqrt{k_x m}\right)^2\right]^{1/2}}{\Delta(\omega^2)} \qquad 38$$

And

$$A_\varphi = \frac{P_x L\,\omega_{nx}\left(\omega_{nx}^2 + 4\xi_x\omega^2\right)^{1/2}}{M_m\,\Delta\omega^2} \qquad 39$$

In case the footing is subjected to the action of a moment and a horizontal force, the resulting amplitudes of sliding and rocking may be obtained by adding the corresponding solutions from Eqs.35, 36, 38 and 39.

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Embedded Foundation

Table 3. Value of equivalent spring and damping constants for embedded foundations (Beredugo and Novak 1972, Novak and Beredugo 1972, Novak and Sachs 1973)

Mode of Vibration	Equivalent spring	Equivalent Damping constant	Damping ratio	
Vertical	$k_{ze} = Gr_o\left[\bar{C}_1 + \frac{G_s}{G}\frac{h}{r_o}\bar{S}_1\right]$	$c_{ze} = r_o^2\sqrt{\rho G}\left(\bar{C}_2 + \bar{S}_2\frac{h}{r_o}\sqrt{\frac{\rho_s}{\rho}\frac{G_z}{G}}\right)$	$\xi_{ze} = \frac{c_{ze}}{2m\omega_{nze}}$	The values of frequency independent parameters $\bar{s}$'s for the elastic space are given in Table 4.
Sliding	$k_{xe} = Gr_o\left[\bar{C}_{x1} + \frac{G_s}{G}\frac{h}{r_o}\bar{S}_{x1}\right]$	$c_{xe} = \sqrt{G\rho}r_o^2\left(\bar{C}_{x2} + \frac{h}{r_o}\sqrt{\frac{\rho_z}{\rho}\frac{G_z}{G}}\bar{S}_{x2}\right)$	$\xi_{xe} = \frac{c_x}{2m\omega_{nxe}}$	The values of frequency independent parameters $\bar{s}$'s for the elastic space are given in Table 4.
Rocking	$k_{\phi e} = Gr_o^3\left\{\bar{C}_{\phi 1} + \frac{G_s}{G}\left(\frac{h}{r_o}\right)\left(\bar{S}_{\phi 1} + \frac{h^2}{3r_o^2}\bar{S}_{x1}\right)\right\}$	$c_{\phi e} = \sqrt{\rho G}r_o^4\left\{\bar{C}_{\phi 2} + \frac{G_s}{G}\frac{h}{r_o}\left(\bar{S}_{\phi 2} + \frac{1}{3}\frac{h^2}{r_o^2}\bar{S}_{x2}\right)\right\}$	$\xi_{\phi e} = \frac{c_\phi}{2M_{mo}m\omega_{n\phi e}}$	r_o and h refer to radius and depth of embedment of the foundation respectively
Torsional or Yawing	$k_{\psi e} = Gr_o^3\left(\bar{C}_{\psi 1} + \left(\frac{G_s}{G}\frac{h}{r_o}\right)\bar{S}_{\psi 1}\right)$	$c_{\psi e} = r_o^4\sqrt{\rho G}\left[\bar{C}_{\psi 2} + \bar{S}_{\psi 2}\frac{h}{r_o}\sqrt{\frac{\rho_s}{\rho}\frac{G_s}{G}}\right]$	$\xi_{\psi e} = \frac{c_\psi}{2M_{mz}m\omega_{n\psi e}}$	

Table 4. ***Values of elastic half-space and side layer parameters for embedded foundations (Beredugo and Novak 1972, Novak and Beredugo 1972, Novak and Sachs 1973)***

Mode of vibration	Poisson's ratio ν	Elastic half-space: Frequency-independent constant parameter	Elastic half-space: Validity range	Side layer: Frequency-independent constant parameter	Side layer: Validity range
Vertical	0.0	$\bar{C}_1 = 3.90$ $\bar{C}_2 = 3.50$	$0 \le a_0 \le 1.5$ (for all values of ν)	$\bar{S}_1 = 2.70$ $\bar{S}_2 = 6.7$ (for all values of ν)	$0 \le a_0 \le 1.5$ (for all values of ν)
	0.25	$\bar{C}_1 = 5.20$ $\bar{C}_2 = 5.00$			
	0.5	$\bar{C}_1 = 7.50$ $\bar{C}_2 = 6.80$			
Sliding	0	$\bar{C}_{x1} = 4.30$ $\bar{C}_{x2} = 2.70$	$0 \le a_0 \le 2.0$	$\bar{S}_{x1} = 3.60$ $\bar{S}_{x2} = 8.20$	$0 \le a_0 \le 1.5$
	0.25			$\bar{S}_{x1} = 4.00$ $\bar{S}_{x2} = 9.10$	$0 \le a_0 \le 2.0$ $0 \le a_0 \le 1.5$
	0.4			$\bar{S}_{x1} = 4.10$ $\bar{S}_{x2} = 10.60$	$0 \le a_0 \le 2.0$ $0 \le a_0 \le 1.5$
	0.5	$\bar{C}_{x1} = 5.10$ $\bar{C}_{x2} = 0.43$	$0 \le a_0 \le 2.0$		
Rocking	0	$\bar{C}_{\phi 1} = 2.50$ $\bar{C}_{\phi 2} = 0.43$	$0 \le a_0 \le 1.0$	$\bar{S}_{\phi 1} = 2.50$ $\bar{S}_{\phi 2} = 1.80$ (for any value of ν)	$0 \le a_0 \le 1.5$
Torsional or yawing	Any value	$\bar{C}_{\psi 1} = 4.3$ $\bar{C}_{\psi 2} = 0.7$	$0 \le a_0 \le 2.0$	$\bar{S}_{\psi 1} = 12.4$ $\bar{S}_{\psi 1} = 10.2$ $\bar{S}_{\psi 2} = 2.0$ $\bar{S}_{\psi 2} = 5.4$	$0 \le a_0 \le 2.0$ $0.2 \le a_0 \le 2.0$ $0 \le a_0 \le 2.0$ $0.2 \le a_0 \le 2.0$

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Table 5 ***Computation response of an embedded foundation by elastic half-space method for coupled rocking and sliding (Beredugo and Novak 1972)***

The values of parameters $\bar{C}_{x1}, \bar{C}_{x2}, \bar{C}_{\phi 1}, \bar{C}_{\phi 2}, \bar{S}_{x1}, \bar{S}_{x2}, \bar{S}_{\phi 1},$ *and* $\bar{S}_{\phi 2}$ are given in Table 4.

L is the height of the centre of gravity above the base.

The horizontal force P_x and the moment M_y act at the centre of gravity of the foundation.

The equations given in this table are used for coupled rocking and sliding of embedded foundations only.

S_{ry} = rocking impedance (moment-rotation ratio), for rotational motion about the short centroidal axis (y) of the foundation basement; and S_T= torsional impedance (moment-rotation ratio), for rotational oscillation about the vertical axis (z).

Item	Equation
Stiffness in sliding	$k_{xe} = Gr_o\left(\bar{C}_{x1} + \frac{G_s}{G}\frac{h}{r_o}\bar{S}_{x1}\right)$
Stiffness in rocking	$k_{\phi e} = Gr_o^3\left\{\bar{C}_{\phi 1} + \left(\frac{L}{r_o}\right)^2 \bar{C}_{x1} + \frac{G_s}{G}\left(\frac{h}{r_o}\right)\bar{S}_{\phi 1} + \frac{G_s}{G}\left(\frac{h}{r_o}\right)\times\left[\left(\frac{h^2}{3r_o^2} + \frac{L^2}{r_o^2} - \frac{hL}{r_o^2}\right)\bar{S}_{x1}\right]\right\}$
Cross-coupling stiffness	$k_{x\phi e} = -Gr_o\left\{L\bar{C}_{x1} + \frac{G_s}{G}\left(\frac{h}{r_o}\right)\left(L - \frac{h}{2}\right)\bar{S}_{x1}\right\}$
Damping constant in sliding	$c_{xe} = \sqrt{\rho G r_o^2}\left(\bar{C}_{x2} + \left(\frac{h}{r_o}\right)\sqrt{\frac{\rho_s}{\rho}\frac{G_s}{G}}\bar{S}_{x2}\right)$
Damping constant in rocking	$c_{\phi e} = \sqrt{\rho G r_o^4}\left\{\bar{C}_{\phi 2} + \left(\frac{L}{r_o}\right)^2 \bar{C}_{x2} + \left(\frac{h}{r_o}\right) + \left(\frac{h}{r_o}\right)\sqrt{\frac{\rho_s}{\rho}\frac{G_s}{G}} \times \left[\bar{S}_{\phi 2} + \left(\frac{h^2}{3r_o^2} + \frac{L^2}{r_o^2} - \frac{hL}{r_o^2}\right)\bar{S}_{x2}\right.\right.$
Cross-coupling damping	$c_{x\phi e} = -\sqrt{\rho G r_o^2}\left[L\bar{C}_{x2} + \left(\frac{h}{r_o}\right)\sqrt{\frac{\rho_s}{\rho}\frac{G_s}{G}}\left(L - \frac{h}{2}\right)\bar{S}_{x2}\right]$
Frequency equation	$\left(k_{xe} - m\omega_n^2\right)\left(k_{\phi e} - M_m\omega_n^2\right) - k_{x\phi e}^2 = 0$
Amplitude in sliding (damped)	$A_{xe} = P_x\sqrt{\frac{\alpha_1^2 + \alpha_2^2}{\varepsilon_1^2 + \varepsilon_2^2}}$
Amplitude in rocking (damped)	$A_{\phi e} = M_y\sqrt{\frac{\beta_1^2 + \beta_2^2}{\varepsilon_1^2 + \varepsilon_2^2}}$
Various terms in equations for A_{xe} *and* $A_{\phi e}$	$\alpha_1 = k_{\phi e} - M_m\omega^2 - \left(\frac{M_y}{P_x}\right)k_{x\phi e}$ $\alpha_2 = \left(c_{\phi e} - \frac{M_y}{P_x}c_{x\phi e}\right)\omega$ $\beta_1 = k_{xe} - m\omega^2 - \frac{P_x}{M_y}k_{x\phi e}$ $\quad \beta_2 = \left(c_{xe} - \frac{P_x}{M_y}c_{x\phi e}\right)\omega$ $\varepsilon_1 = mM_m\omega^4 - \left[mk_{\phi e} + M_m k_{xe} + c_{xe}c_{\phi e} - c_{x\phi e}^2\right]\omega^2 + \left[k_{xe}k_{\phi e} - k_{x\phi e}^2\right]$ $\varepsilon_2 = -\left[mc_{\phi e} + M_m c_{xe}\right]\omega^3 + \left[c_{xe}k_{\phi e} + c_{\phi e}k_{xe} - 2c_{x\phi e}k_{x\phi e}\right]\omega$

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CONCLUSION

- Dynamic loads are frequency dependent.
- During Earthquake, frequency of ground motion is time dependent.
- Liquefaction is major type of damage in saturated soft soils.
- Liquefaction may occur at same site in more than one event.
- Double amplification was observed characteristically during 1985 Mexico earthquake.
- More Importantly –New events may teach us new knowledge which may be used for safer design of safer structures.

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QUESTIONS ?

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